4.3 Solving Polynomial Inequalities

A Sign Chart Method

Use a *sign chart* to specify the sign of each factor and then combine them to find the sign of the whole factored polynomial.

Ex 1. Use a sign chart to solve each inequality.

a)
$$x(x-1) \le 0$$

b)
$$(x^2 - x - 2)(x^2 + 1)(x - 3)^3 \ge 0$$

B Substitution Method

To find the sign of the polynomial expression on each interval, chose conveniently a number in that interval and evaluate the function.

Ex 2. Use a substitution to solve the following inequality.

$$x^2(x^2-4)(x+1) > 0$$

C Graphical Method

Graph the factored polynomial and then conclude about its sign.

Ex 3. Use the graphical method to solve each inequality.

a)
$$x(x-1)(x+2) > 0$$

b)
$$(x^4 - 1)(x^2 - 9) \le 0$$

D Algorithm to Solve Polynomial Inequalities

In order to solve an inequality involving a polynomial expression:

- Move all terms to one side of inequality
- Factor the polynomial
- Use the sign chart or the graphical method to find the sign of the polynomial
- Write the solution set

Ex 4. Solve for x.

a)
$$(x+1)^2 \le (x-2)^3 + 5$$

b)
$$(x-1)^4 + 8x > 4(x^2 + 1)$$

E Technology

When the polynomial inequality is not factorable, use technology to find the solution set.

Ex 5. A box with an open top is to be constructed from a square piece of cardboard, 2 m wide, by cutting out a square, of side length x, from each of the four corners and bending up the sides. Find x such that the volume of the box is between $0.3 m^3$ and $0.5 m^3$.

Reading: Nelson Textbook, Pages 219-225